Exoplanetary Microlensing

Joseph Catanzarite
Jet Propulsion Laboratory,
California Institute of Technology
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With material from Dave Bennett, Scott Gaudi, and others.

2011 Sagan Exoplanet Summer Workshop

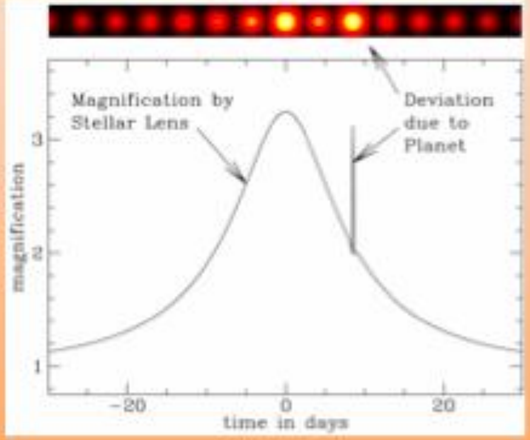
Exploring Exoplanets with Microlensing

July 25-29, 2011, California Institute of Technology

June 7, 2011: Early Registration Fee deadline

Topics include:

- History of Microlensing, Theory, Detection and Follow-up
- Introduction to Microlensing Photometric Techniques
- HST/AO Data Reduction
- Microlensing with Space-based Telescopes
- Modeling of Microlensing Data
- Extracting the Physical Parameters of Planetary Events
- Null Results and Detection Efficiency
- Future Prospects and Challenges of Microlensing



Hands-on Sessions during the week will allow attendees to work with microlensing data.

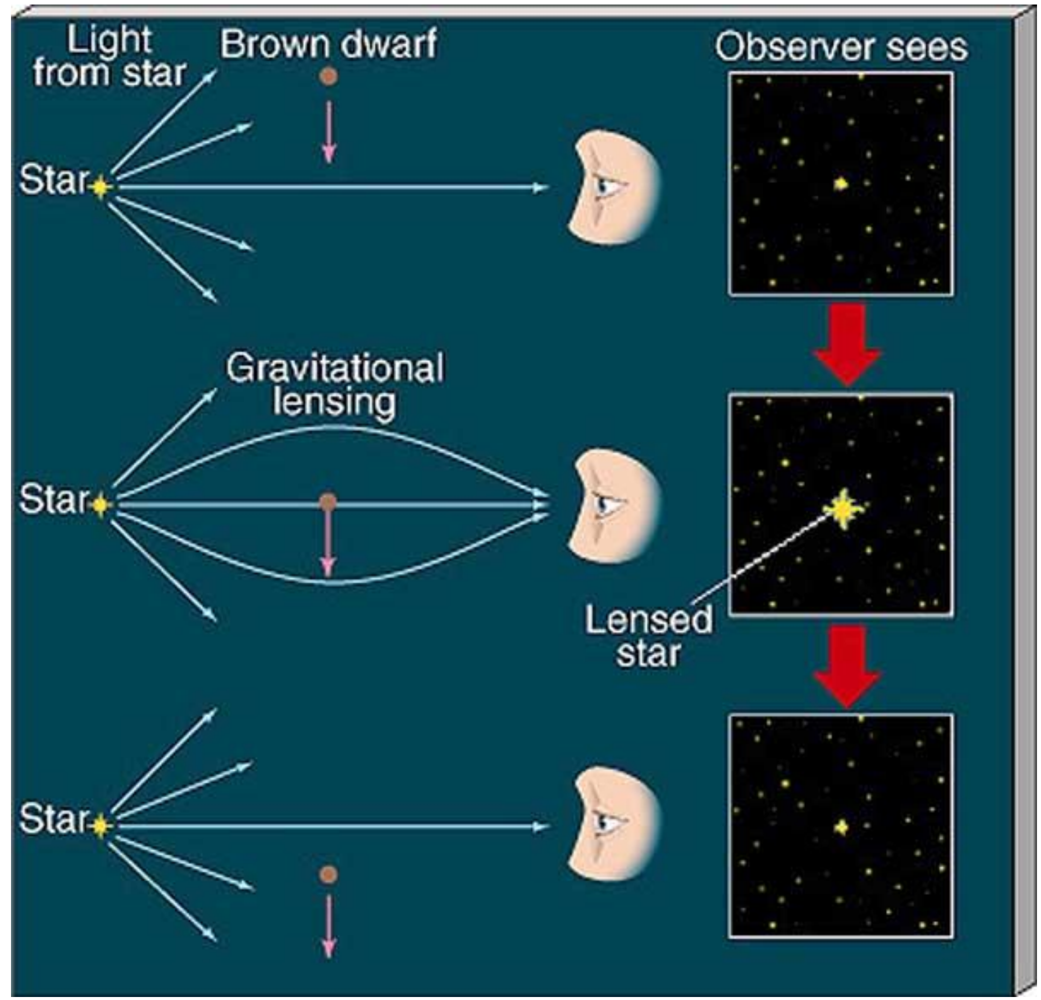
Scientific Organizing Committee

Dave Bennett (University of Notre Dame)
Ian Bond (Massey University, New Zealand)
Subo Dong (Institute for Advanced Study)
Scott Gaudi (Ohio State University)
Stephen Kane (NExScI)
Rachel Street (LCOGT)
Takahiro Sumi (Nagoya University)

http://nexsci.caltech.edu/workshop/2011

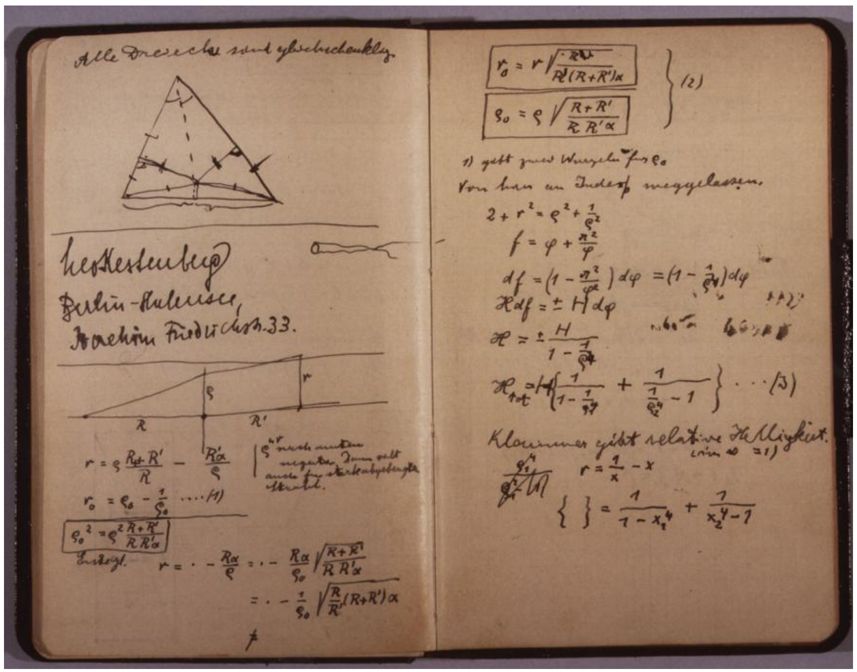
Part 1. Microlensing Phenomenology

- **When a foreground star crosses near a background star, its gravity bends and focuses the light from the background star.**
- **We call the background star the *source*, and the foreground star the *lens*.**



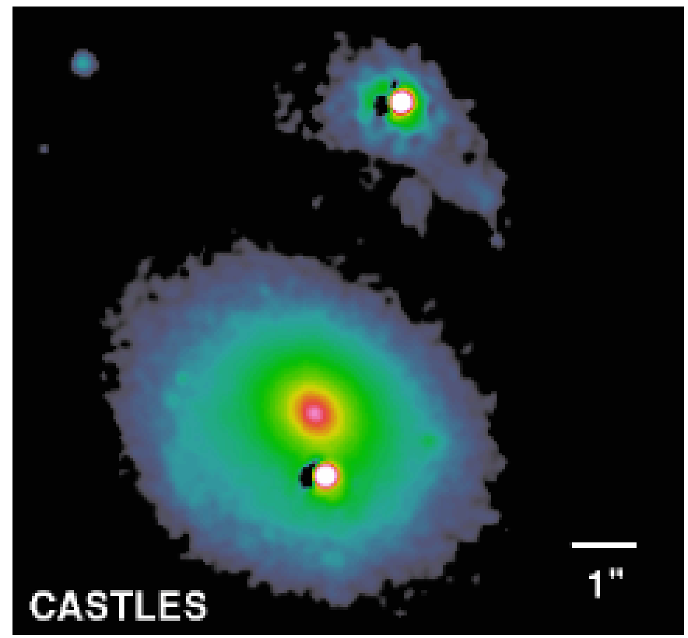
Einstein predicted gravitational lensing 100 years ago

Einstein's Notebook entry from 1912



He published similar calculations in Science magazine in 1936.

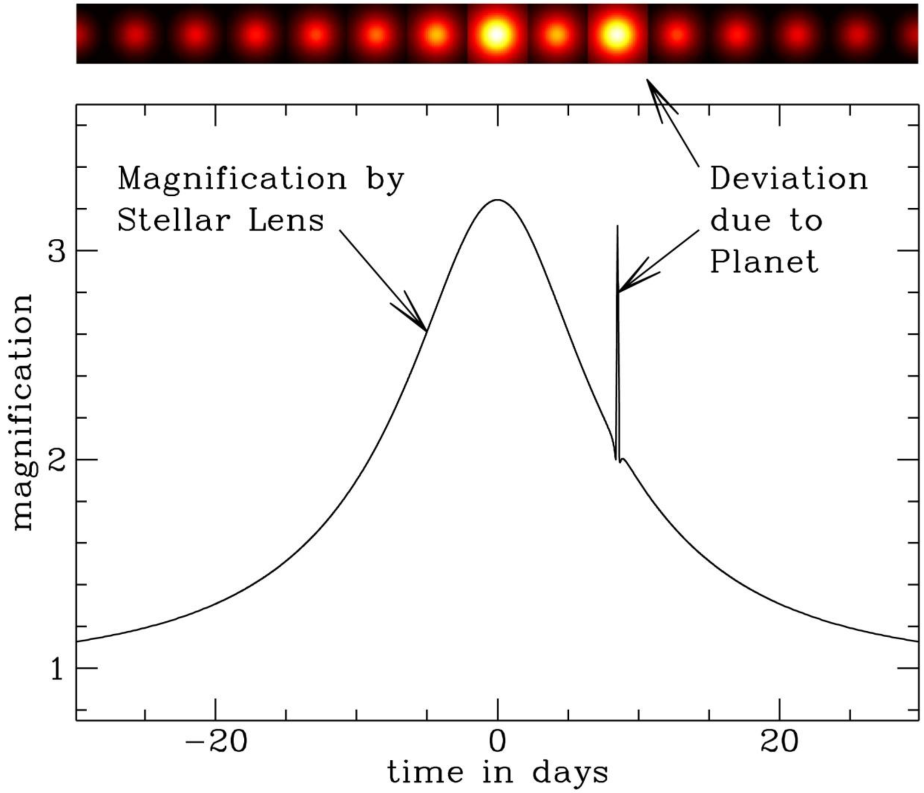
Confirmation in 1979



Double quasar Q0957+561 A/B turned out to be 2 images of the same quasar , lensed by a foreground galaxy.

Microlensing event light curve

- **Stellar microlensing occurs when a foreground star's gravity bends light from a background star.**
- **The *source* (background star) is observed to brighten as the *lens* (foreground star) approaches.**
- **The source brightness peaks at minimum separation, and dims back to normal after the lens passes by.**
- **The typical duration of a stellar event is about a month.**
- **If the lens star has a planet, its gravity may cause a deviation in the stellar light curve, lasting hours to days.**



[Movie of star/planet microlensing]()

The light curve (magnification vs. time) is the essential observable.

"I don't understand. You are looking for planets you can't see around stars you can't see."

Debra Fischer
RV planet hunter, at
2000 Microlensing
Workshop

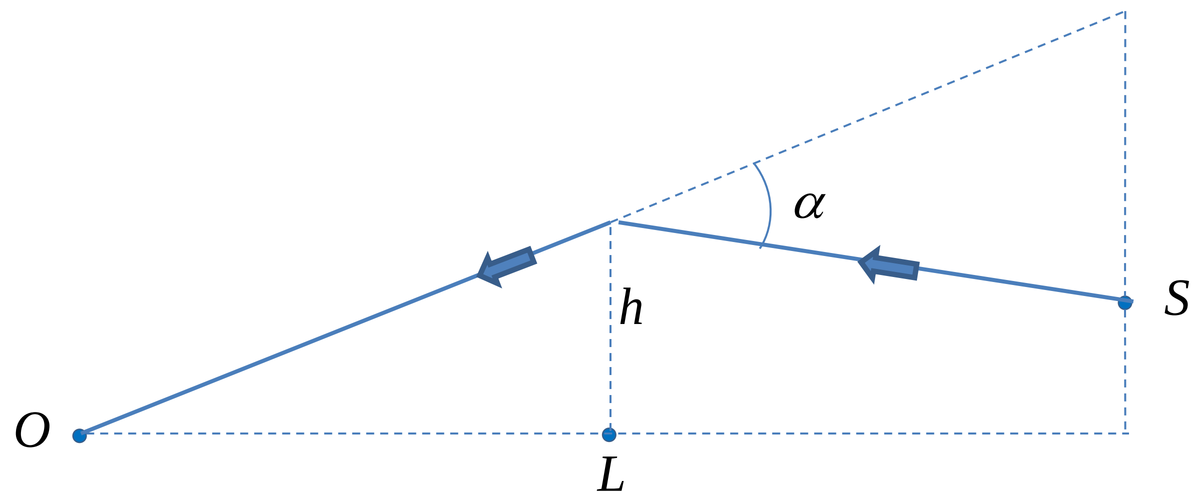
Part 2. Geometry of Microlensing

- Gravitational deflection of light
- The lens equation
- Einstein ring
- Einstein radius
- Einstein crossing time
- Point source + point lens
  - Images
  - Magnification
- Multiple sources
  - Critical curves
  - Caustics

Gravitational Deflection of Light

- According to General Relativity, mass bends light
- The **gravitational deflection angle** α of a light ray due to a point lens of mass *M* is

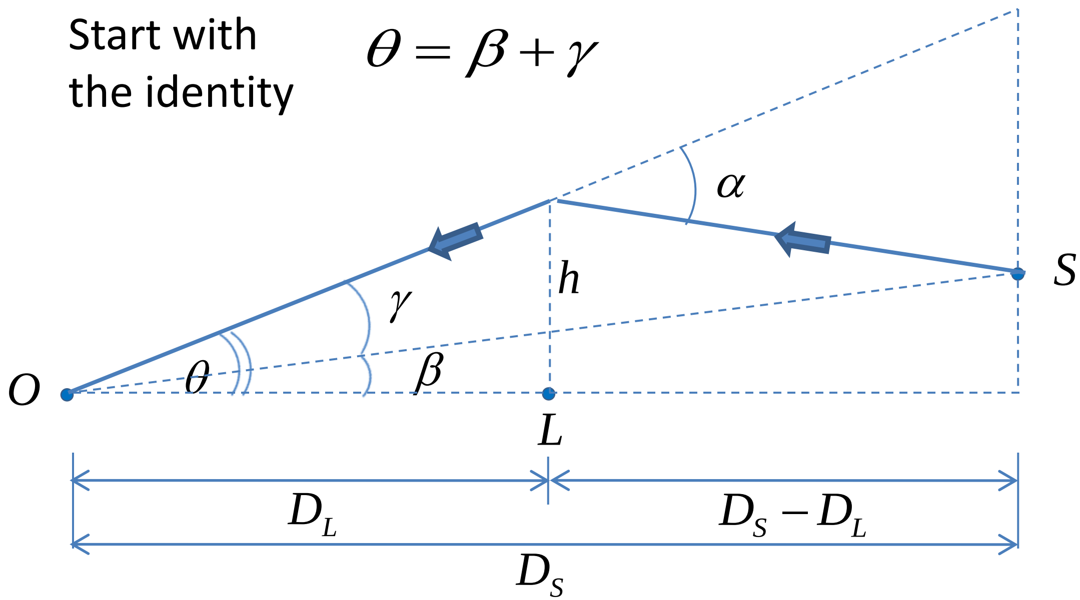
$$\boxed{\alpha = \frac{4GM}{hc^2}}$$



- O, L, and S are observer, lens, and source
- *h* is the distance from the lens mass *M* to the intersection of the undeflected light ray with the lens plane.

The Lens Equation

Start with the identity $\theta = \beta + \gamma$



O, L, S: observer, lens and source

$\alpha = \dfrac{4GM}{hc^2}$: deflection angle.

D_L, D_S : distances from O to L and S.

θ, β : image and source positions w.r.t. line between O and L.

γ : angle between image and source.

Relation between γ and α follows from the small-angle approximation:

$$\gamma = \frac{D_S - D_L}{D_S}\alpha = \frac{D_S - D_L}{D_S D_L}\frac{4GM}{c^2}\frac{1}{\theta}$$

$h = \theta D_L$ and $\alpha = \dfrac{4GM}{hc^2}$

so $\alpha = \dfrac{4GM}{D_L c^2}\dfrac{1}{\theta}$

Defining $\theta_E \equiv \sqrt{\dfrac{D_S - D_L}{D_S D_L}\dfrac{4GM}{c^2}}$ we have $\gamma = \dfrac{\theta_E^2}{\theta}$ and the **lens equation** is:

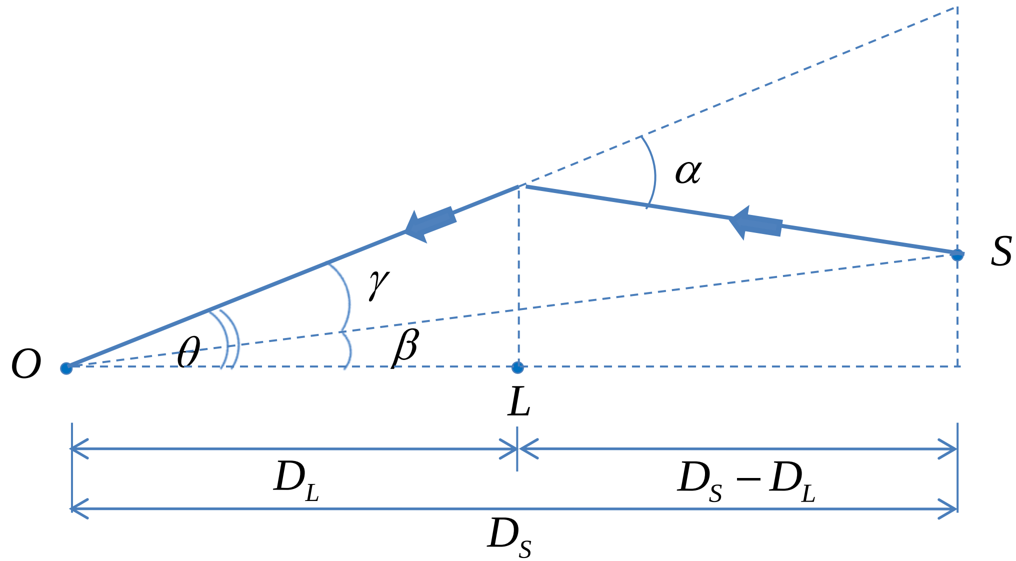
$$\boxed{\theta = \beta + \frac{\theta_E^2}{\theta}}$$

The Einstein radius

If the source and lens are aligned, then

$\beta = 0$ and the lens equation $\theta = \beta + \dfrac{\theta_E^2}{\theta}$

becomes $\quad \theta = \theta_E = \sqrt{\dfrac{D_S - D_L}{D_S D_L} \dfrac{4GM}{c^2}}$



Due to symmetry, the image becomes a circle with angular radius θ_E, called an **Einstein ring.**

θ_E is the lensing star's gravitational zone of influence. Typically a few milliarcsec, it sets the angular scale of microlensing.

$$\boxed{\theta_E \approx 0.55 mas \sqrt{\frac{1 - D_L/D_S}{D_L/D_S}} \left(\frac{D_S}{8kpc}\right)^{-1/2} \left(\frac{M}{0.3 M_{Sun}}\right)^{1/2}}$$

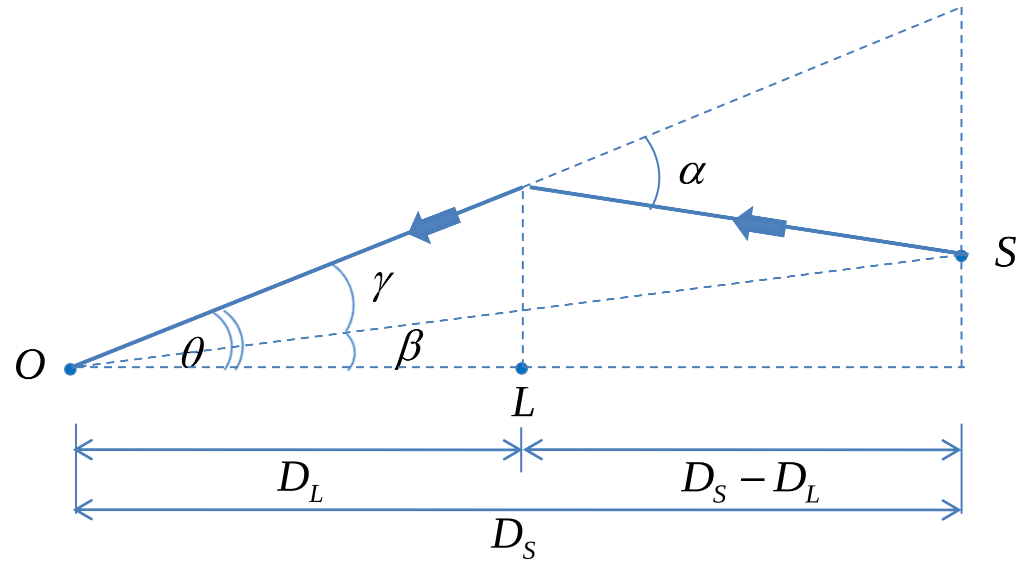
The physical radius of the circle in the 'lens plane' is the **Einstein radius** $\quad R_E = \theta_E D_L$

$$\boxed{R_E \approx 2.2 AU \sqrt{4(D_L/D_S)(1 - D_L/D_S)} \left(\frac{D_S}{8kpc}\right)^{1/2} \left(\frac{M}{0.3 M_{Sun}}\right)^{1/2}}$$

Images

Lens equation

$$\theta = \beta + \frac{\theta_E^2}{\theta}$$



Scale the source and image angular positions by the Einstein angle

$$u \equiv \frac{\beta}{\theta_E} \qquad y \equiv \frac{\theta}{\theta_E}$$

u and y are the angular separations between lens and source and lens and image, in units of Einstein angle.

Rewrite the lens equation in terms of the scaled angles

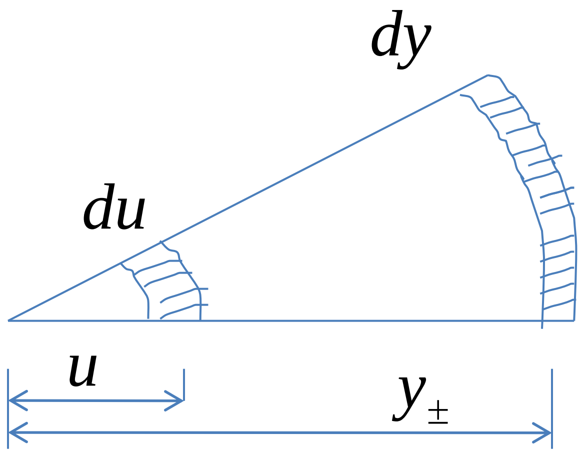
$$y = u + \frac{1}{y}$$

The lens equation is quadratic ; solution yields two images:

$$y_\pm = \pm\frac{1}{2}\left(\sqrt{u^2+4} \pm u\right)$$

Magnification

Surface brightness is conserved, so the magnification is the ratio of the image area to the source area:



$$A_\pm = \left| \frac{y_\pm}{u} \frac{dy_\pm}{du} \right| = \frac{1}{2}\left[\frac{u^2+2}{u\sqrt{u^2+4}} \pm 1 \right]$$

The images are unresolved, so we observe only the total magnification

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$u << 1 \qquad A \approx \frac{1}{u} \propto \sqrt{M_L}$

$u = 1 \qquad A = 1.34$

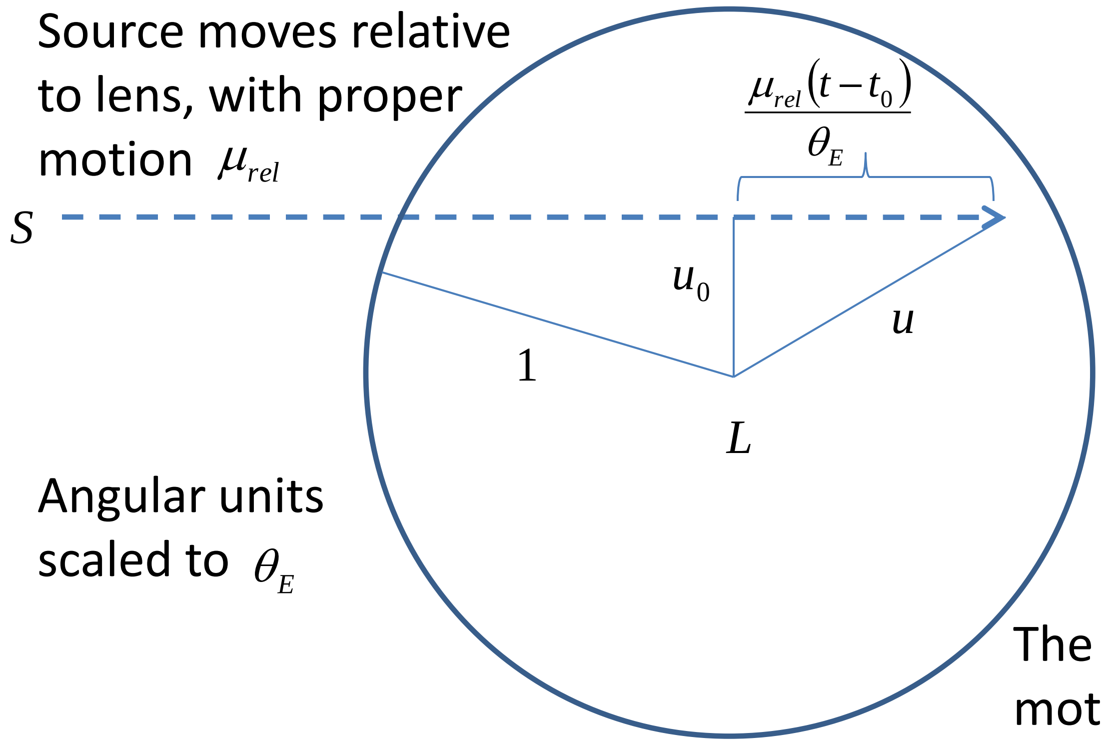
$u >> 1 \qquad A \approx 1 + \frac{2}{u^4}$

When S and L are aligned (Einstein ring case),

$u = 0$ and magnification becomes infinite.

Time dependence of source motion

Source moves relative to lens, with proper motion μ_{rel}



Angular units scaled to θ_E

$$u(t) = \sqrt{\frac{\mu_{rel}{}^2}{\theta_E{}^2}(t-t_0)^2 + u_0^2}$$

Define the **Einstein crossing time**, the time for the source traverse the Einstein radius: $t_E \equiv \frac{\theta_E}{\mu_{rel}}$

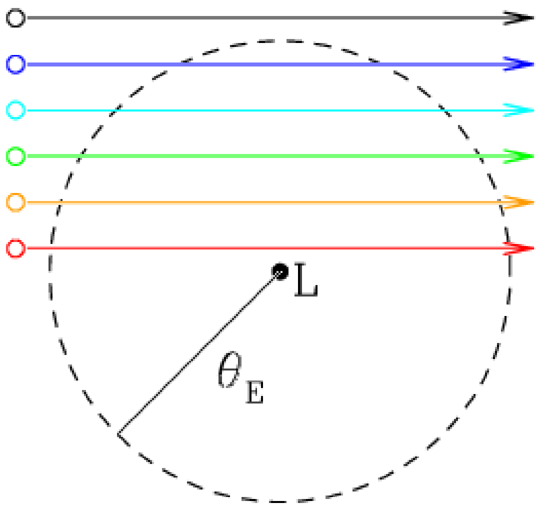
The time dependence of the source motion is:

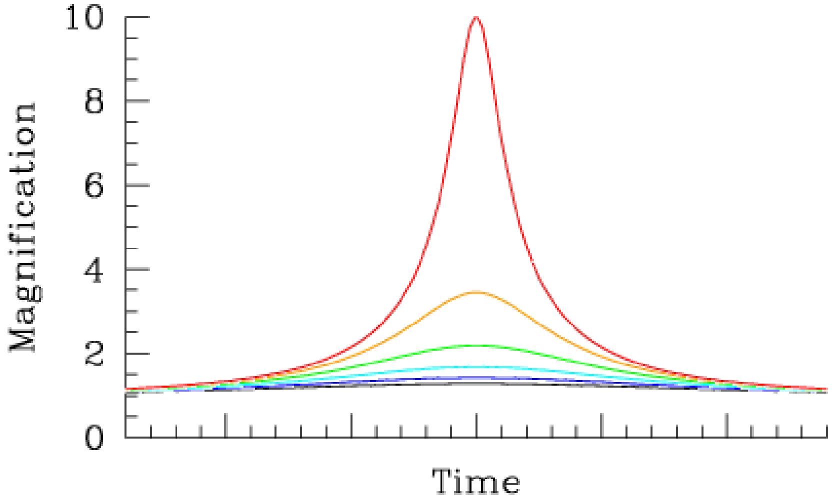
$$u(t) = \sqrt{\frac{(t-t_0)^2 + u_0^2}{t_E{}^2}}$$

u_0 is the angular distance of closest approach between the source and the lens, called the **impact parameter**.

$$t_E = 19 day \sqrt{4\frac{D_L}{D_S}\left(1-\frac{D_L}{D_S}\right)}\left(\frac{D_S}{8kpc}\right)^{1/2}\left(\frac{M_L}{0.3M_{Sun}}\right)^{1/2}\left(\frac{\mu_{rel}D_L}{200km/s}\right)$$

Time dependence of magnification





The impact parameter is the distance from a source trajectory (colored lines) to the lens L.

$$u(t) = \sqrt{\frac{(t - t_0)^2 + u_0^2}{t_E^{\ 2}}}$$

The time dependence of the magnification is:

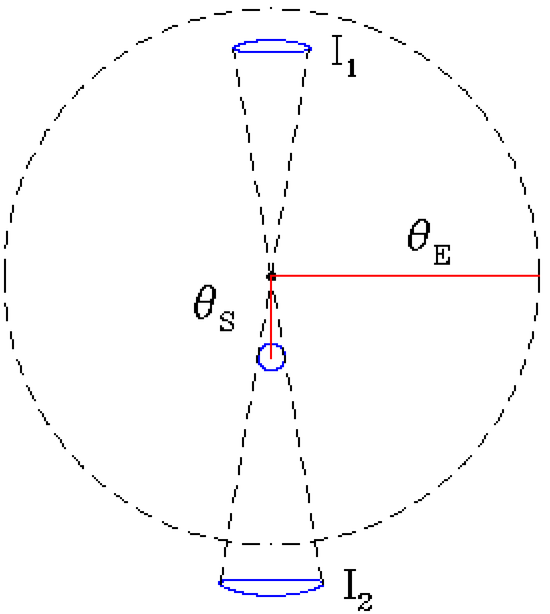
$$A(t) = \frac{u(t)^2 + 2}{u(t)\sqrt{u(t)^2 + 4}}$$

It turns out that the width of the magnification is proportional to the square root of the mass of the lens.

Major and minor images

$$\theta_E = \sqrt{\frac{4GM_L}{c^2}\frac{D_S - D_L}{D_L D_S}}$$

- θ_E is the angular Einstein radius
  - D_S and D_L are the distances to the source and lens stars
  - M_L is the mass of the lens star
- I_1 is the minor image, inside the Einstein radius.
- I_2 is the major image , outside the Einstein radius.
- θ_S is the angular distance between source (blue circle) and lens (black dot).
- The images are unresolved; only the sum of their flux is observed.



Lens equation for N point lenses

- Consider N point lenses, at positions $\vec{r}_k = (x_k, y_k)$, with mass , total mass $M = \sum_{k=1}^{N} M_k$ where the $\vec{r}_k$ are normalized to the Einstein angle.

- The lens equation for a point lens generalizes to

$$\vec{r}_s = \vec{r} - \frac{1}{M} \sum_{k=1}^{N} M_k \frac{\vec{r} - \vec{r}_k}{\left|\vec{r} - \vec{r}_k\right|^2}$$

- Complex number representation $z = x + iy$

$$z_s = z - \frac{1}{M} \sum_{k=1}^{N} M_k \frac{z - z_k}{(z - z_k)(\bar{z} - \bar{z}_k)} = z - \frac{1}{M} \sum_{k=1}^{N} \frac{M_k}{(\bar{z} - \bar{z}_k)}$$

- Mapping between images and source

- Complex polynomial of degree $2N^2 + 1$, has no analytic solution

Magnification

- Magnification is found by the transformation of areas, given by the inverse of the Jacobian: $A = \frac{1}{J}$

- The Jacobian is $J = \frac{\partial(x_s, y_s)}{\partial(x, y)}$, and it can be shown that $J = 1 - \left| \frac{\partial z_s}{\partial \bar{z}} \right|^2$

- Now $\frac{\partial z_s}{\partial z} = \frac{1}{M} \sum_{k=1}^{N} \frac{M_k}{\left(\bar{z} - \bar{z}_k\right)^2}$ is computed from the lens equation, so that

$$J = 1 - \left| \frac{1}{M} \sum_{k=1}^{N} \frac{M_k}{\left(\bar{z} - \bar{z}_k\right)^2} \right|^2$$

Critical curves

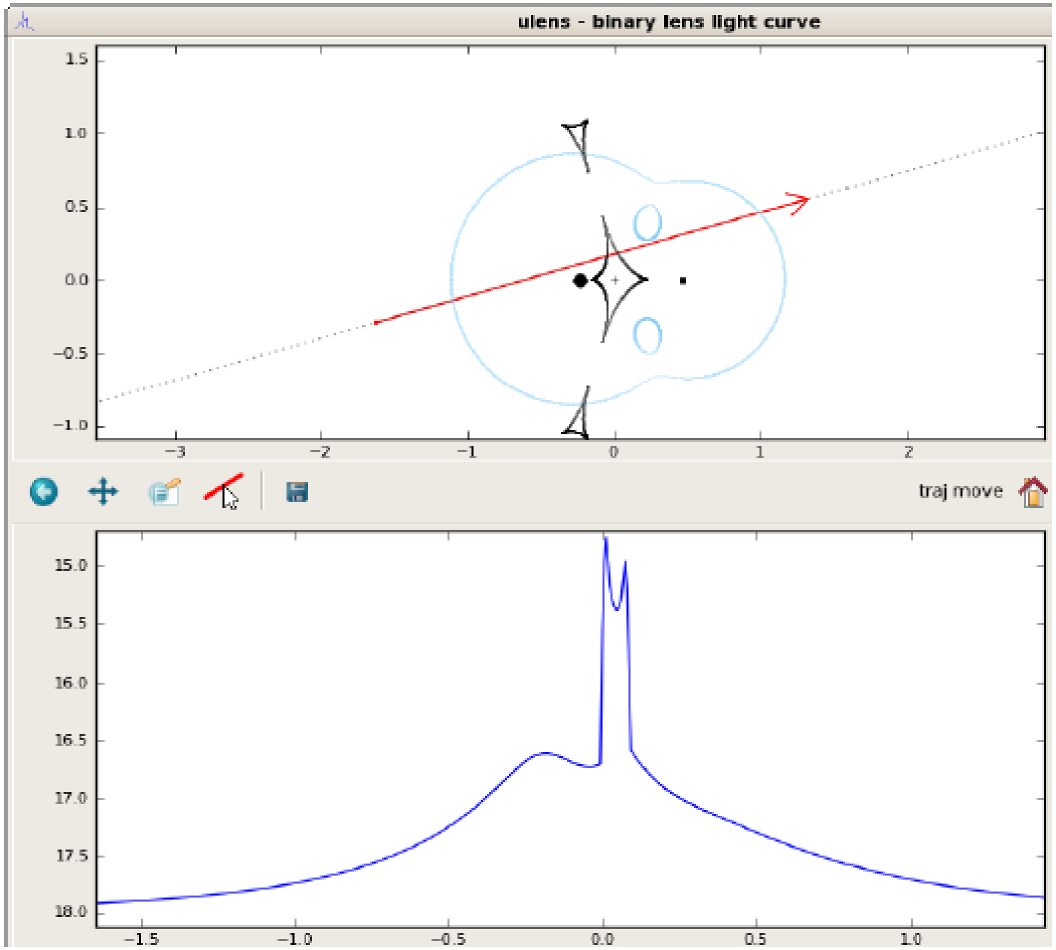
- The Jacobian is $$J = 1 - \left| \frac{1}{M} \sum_{k=1}^{N} \frac{M_k}{(\bar{z} - \bar{z}_k)^2} \right|^2$$
- When J = 0, a point source will be infinitely magnified, and the image positions satisfy $\left| \frac{1}{M} \sum_{k=1}^{N} \frac{M_k}{(\bar{z} - \bar{z}_k)^2} \right|^2 = 1$, or $\frac{1}{M} \sum_{k=1}^{N} \frac{M_k}{(\bar{z} - \bar{z}_k)^2} = e^{i\phi}$

  Where ϕ is a parameter on the interval $[0, 2\pi)$

- Solutions for image positions z form continuous **critical curves**.
- There are at most 2N critical curves.
- The lens equation maps the critical curves into the corresponding source positions, called **caustics**, which are generally continuous closed curves.
- Caustics are source positions with to high magnification – they are places where you hope the source will go.
- For a point lens, the critical curve is the Einstein ring, and the (degenerate) caustic is a point at the position of the lens.

Caustics and critical curve for 2 lens system, star + planet

Lenses (the star and the planet) are the large and small black dots



The source trajectory is in red, critical curves are in light blue, and the caustics are in black.

Magnification curve in blue. Note the high magnification due to the planet during the caustic crossing.

Lensing Optical Depth

- Lensing optical depth is the probability that a source is inside the Einstein radius of a lensing star along the line of sight.
- Alternatively, the fraction of total area on the sky covered by the Einstein angles of all the lenses.
- If the volume density of lensing stars is $n(D_L)$ then the differential lensing optical depth at distance D_L is $d\tau = \frac{1}{4\pi}\left[n(D_L) 4\pi D_L{}^2 dD_L\right]\left(\pi\theta_E{}^2\right)$
- The lensing optical depth for a source at D_S is $\tau = \int_0^{D_S} n(D_L) D_L{}^2 \left(\pi\theta_E{}^2\right) dD_L$
- Based on a simple Milky Way Galaxy model with constant density along the line of sight, one finds $\tau \approx 2.6\times 10^{-7}$

Lensing event rate

A lensing event occurs each time the source moves through an Einstein radius of a lens. For a single source, if each lens moves at constant angular velocity $v_{rel} = D_L \mu_{rel}$ relative to the source, the differential number of lensing events at distance D_L during time t is the area swept out by all the lenses times the surface density of lenses:

$$dn_1 = 2R_E v_{rel} t n(D_L) dD_L = 2\theta_E D_L^2 \mu_{rel} t n(D_L) dD_L = 2\theta_E^2 \frac{t}{t_E} D_L^2 n(D_L) dD_L$$

If there are N_S sources at the same distance being monitored, each with the same average angular motion, (e.g. the galactic bulge) then the total number of events is

$$dn = 2N_S \theta_E^2 \frac{t}{t_E} D_L^2 n(D_L) dD_L$$

Integrating out to the sources gives $$n = 2N_S \theta_E^2 \int_0^{D_S} \frac{t}{t_E} D_L^2 n(D_L) dD_L \approx \frac{2}{\pi} N_S \frac{t}{t_E} \tau$$

assuming all the Einstein crossing times are identical.

The lensing event rate is therefore $$\Gamma \equiv \frac{n}{t} \approx \frac{2N_S}{\pi} \frac{\tau}{t_E} \approx 1200\, yr^{-1} \frac{N_S}{10^8} \frac{\tau}{10^{-6}} \left(\frac{t_E}{19 day} \right)^{-1}$$

The OGLE-III telescope network monitors 2×10^8 stars, and $\Gamma \approx 2400\, yr^{-1}$

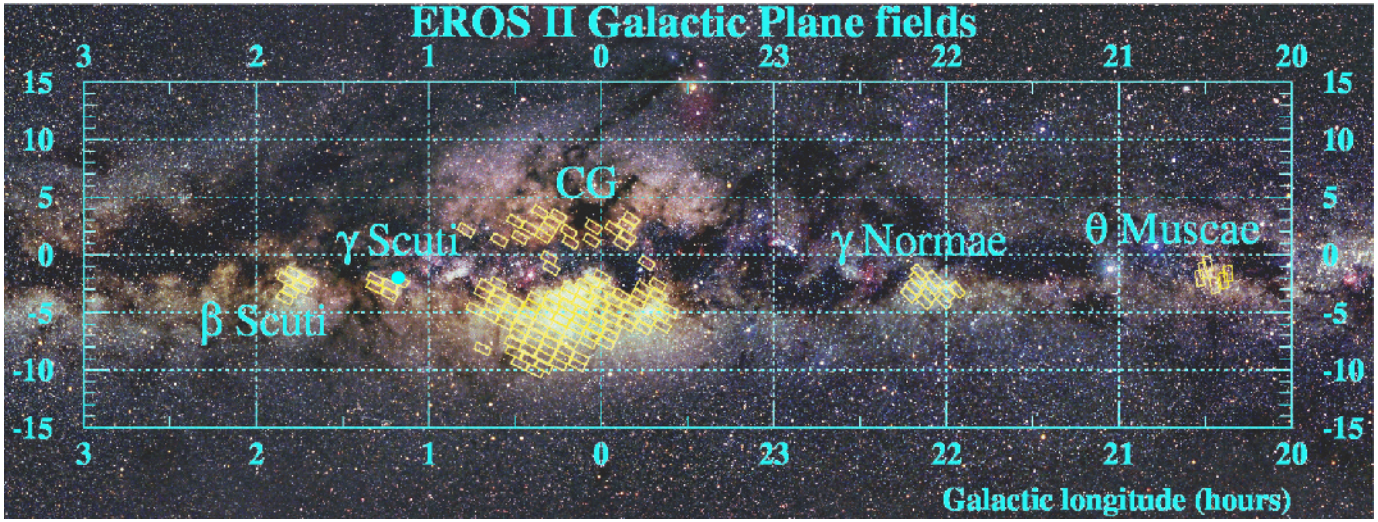
The observed rate is 600 events per year, so the detection efficiency is ~25%

From light curve to planet parameters

- The ‘widths’ of the star and planet peaks in the light curve are proportional to the square roots of the respective masses, so the planet-to-star mass ratio comes from the ratio of widths.
- The time delay of the planet peak with respect to the stellar peak gives the projected separation of the lensing star and its planet, in units of the Einstein radius

$$R_E = D_L \theta_E = D_L \sqrt{\frac{4GM_L}{c^2} \frac{D_S - D_L}{D_L D_S}}$$

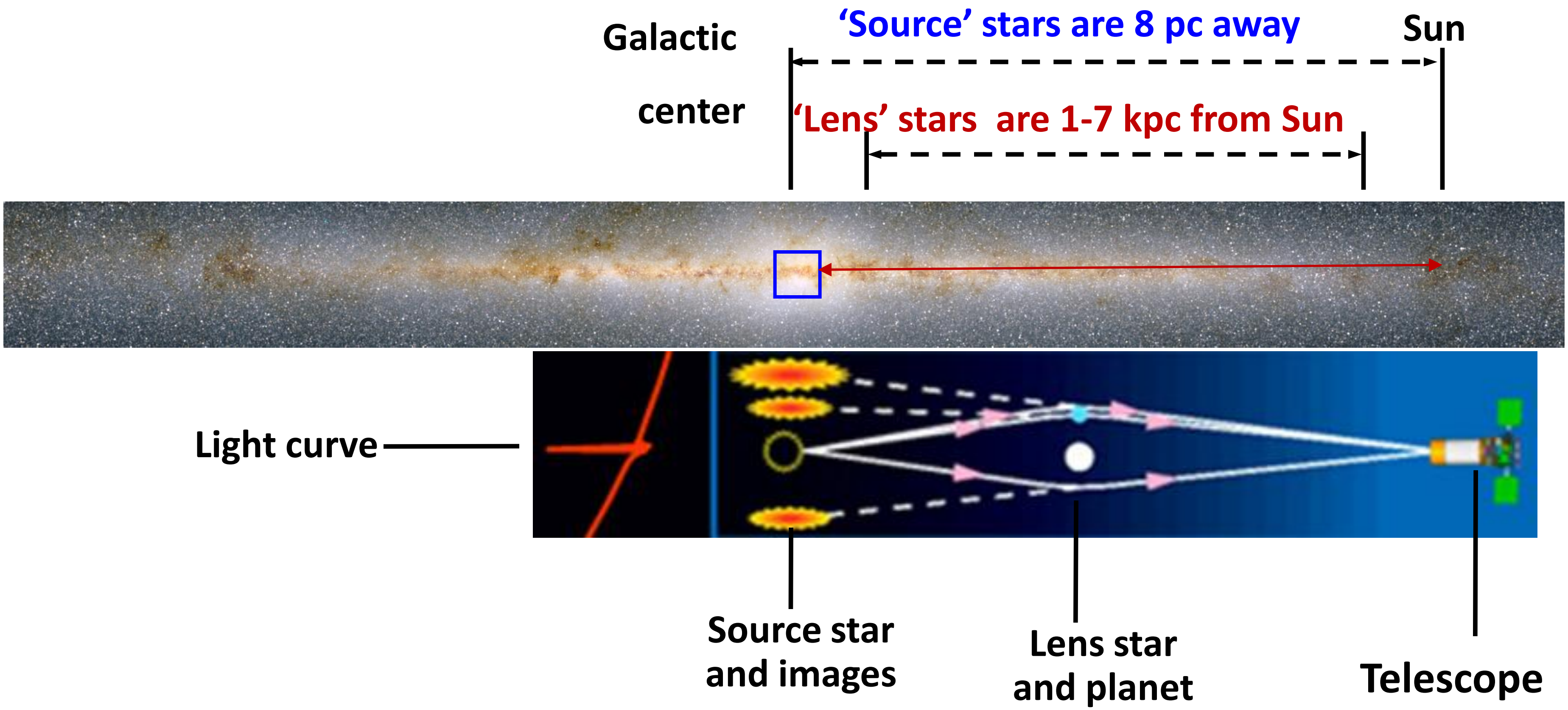
Part 3. Microlensing exoplanet surveys



- **Exoplanetary microlensing is a short- lived, low probability phenomenon**
- **In order to monitor many potential events, we need**
  - **A wide-field survey**
  - **Pointed at a region that is dense in stars, e.g. the galactic bulge**
  - **High-cadence continuous sampling**

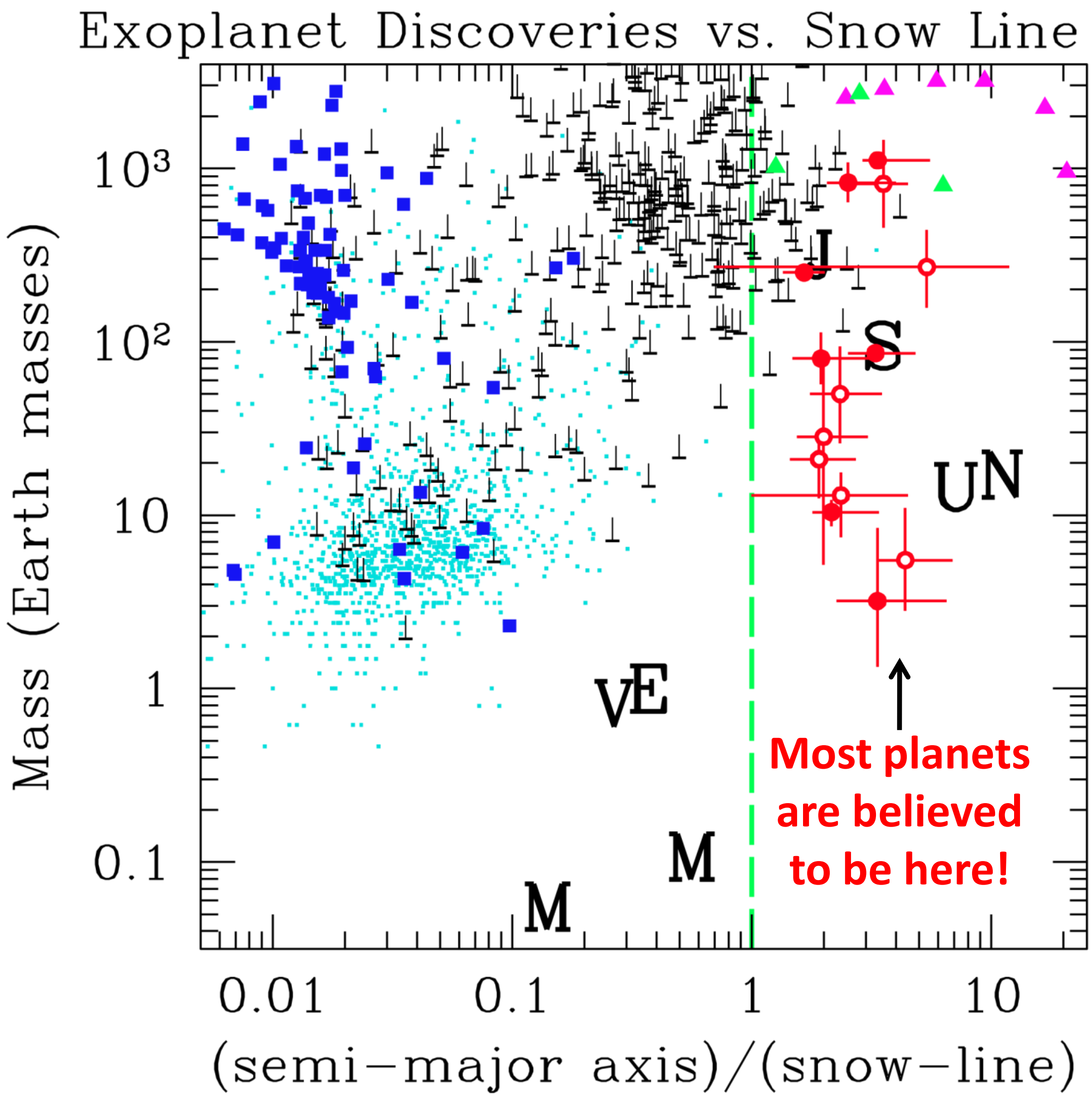
- **Typical 'source' star is a giant or dwarf in the bulge.**
- **Typical 'lens' star is a red main-sequence star in the foreground disk or bulge**

The Best Microlensing Target Fields are in the Galactic Bulge



Strategy: monitor hundreds of millions of 'source' stars in the Galactic bulge in order to detect planetary companions to 'lens' stars in the Galactic disk and bulge.

Exoplanet Science



- Microlensing
- Transit
- Kepler candidates
- Doppler
- Imaged planets

- The ***snow-line*** is defined as 2.7 AU $(M/M_{\odot})$
- Super-Earth planets beyond the snow-line appear to be the most common type yet discovered

13 microlensing planetary systems, one has two planets

Candidates detected by microlensing update : **22 June 2011**

<< Back to the Index Catalog | Data Catalog | Histograms | Correlation Diagrams | Planet Table

(sorted by **increasing period of the closest planet**)

Planet Data (- ALL FORMATS)						
PLANET	M. (M_{Jup}) - stats	RADIUS (R_{Jup}) - stats	PERIOD (days) - stats	SEM-MAJ AXIS (AU) - stats	ECC.	INCL. (deg) - stats
OGLE-2008-BLG-513/MOA-2008-BLG-401 b	6.8	-	-	4	-	-
MOA-2009-BLG-319 b	0.157	-	-	2	-	-
MOA-2008-BLG-310-L b	0.23	-	-	1.25	-	-
MOA-2007-BLG-400-L b	0.9	-	-	0.85	-	-
OGLE-2007-BLG-368L b	0.0694	-	-	3.3	-	-
MOA-2007-BLG-192-L b	0.01	-	-	0.66	-	-
OGLE235-MOA53 b	2.6	-	-	5.1	-	-
OGLE-06-109L b	0.727	-	1790	2.3	-	64
c	0.271	-	4931	4.5	0.15	64
MOA-2009-BLG-387L b	2.6	-	1970	1.8	-	-
MOA-2009-BLG-266L b	0.0327	-	2780	3.2	-	-
OGLE-05-169L b	0.04	-	3300	2.8	-	-
OGLE-05-390L b	0.017	-	3500	2.1	-	-
OGLE-05-071L b	3.5	-	~ 3600	3.6	-	-

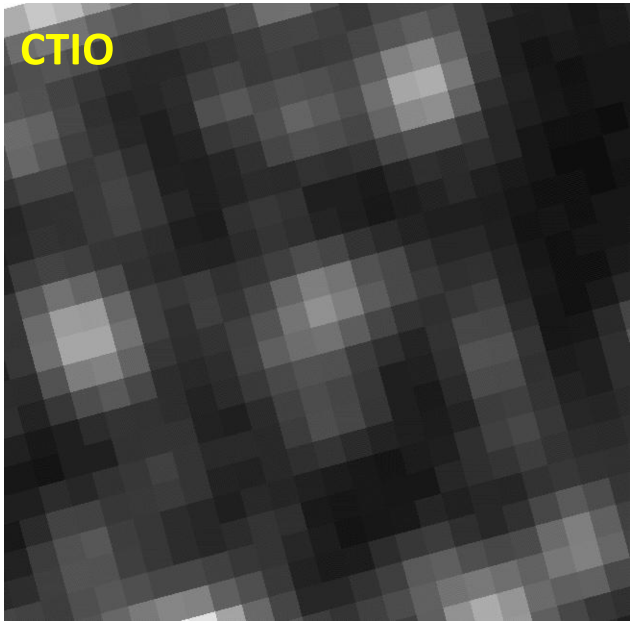
Free-floating planets!

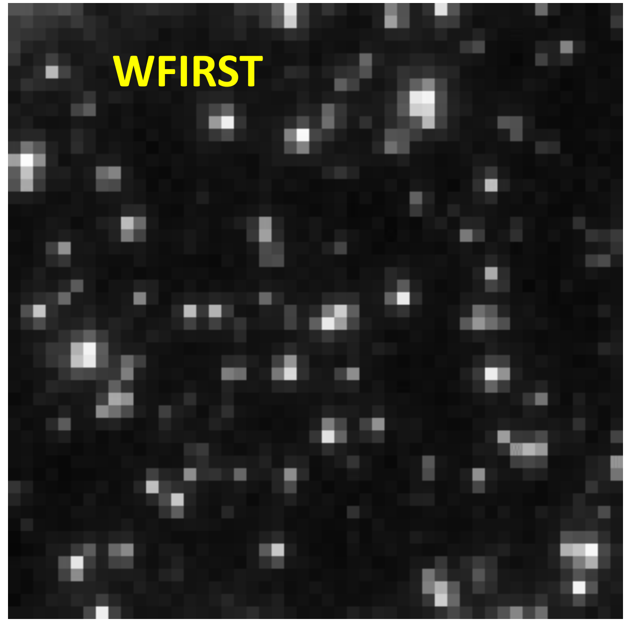
- Bennett et al., Nature, May 2011
- Survey found ten ~Jupiter mass free-floating planets
- Free-floating means no detectable host; planet could be in a wide orbit > 7 to 45 AU
- But direct detection limits from the Gemini Planet Survey say that at most 40% of stars could have a Jupiter-mass planet at 12 AU < a < 500 AU
- Statistical analyis of survey results imply that there are ~2 of these per star!
- Therefore, most of these planets must be unbound

Microlensing provides a statistical census of exoplanets

- Distribution of planet masses and separations, as a function of stellar type
  - Kepler is sensitive to planets down to Earth mass inward of 1 AU.
  - Microlensing is sensitive to planets down to Mars mass, outward of 1 AU.
- Microlensing + Kepler determines the frequency of habitable earth-like planets
- Frequency of free-floating (ejected?) planets
- Frequency of massive moons
- All of these provide important constraints on planet formation theories.

Need a space-based microlensing survey





- **WFIRST microlensing program**
  - **High resolution**
  - **Large FOV**
  - **IR sensitivity**
  - **24 hr duty cycle**

- **Advantages of space-based imaging**
  - **High precision photometry of main sequence source stars**
  - **Lens star detection → absolute planet mass determination**
  - **Sensitive to planets at a wider range of separations**

Wide-Field Infra-Red Survey Telescope

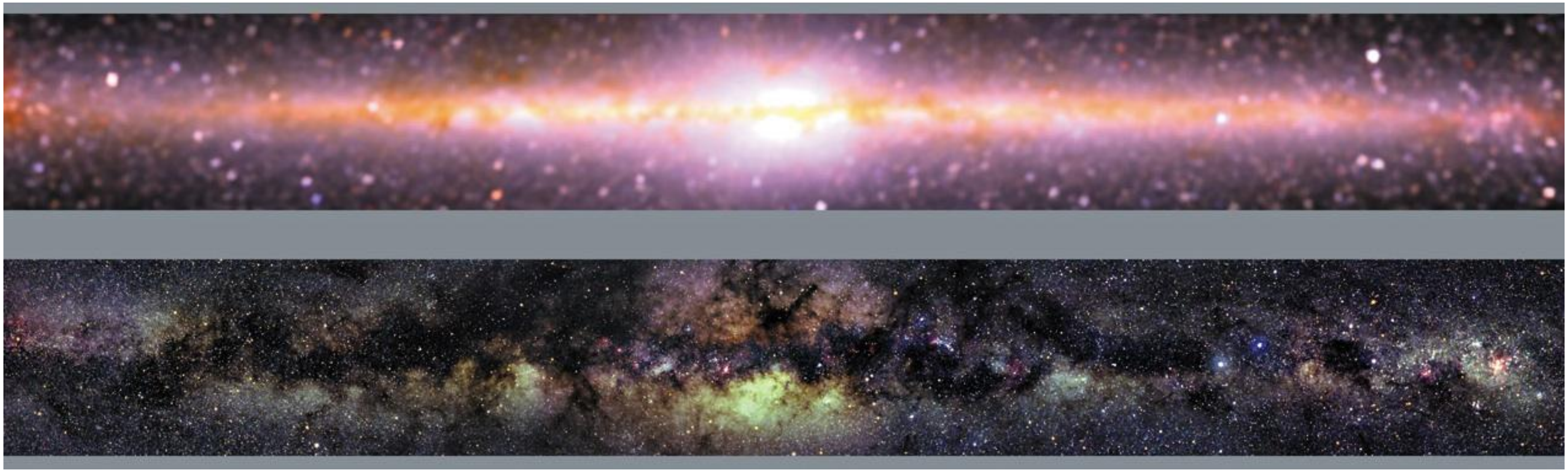
Advantage of observing the lens star

- HST can sometimes resolve the ‘lens’ star, and estimate its distance and mass.
- Knowledge of the lensing star’s mass determines the planet mass from the measurement of the mass ratio M_{planet}/M_{star}
- Knowledge of the lensing star’s mass and distance determine the Einstein radius, allowing an estimate of the absolute projected separation.

Advantages of an infrared survey

The central Milky Way

Near-infrared

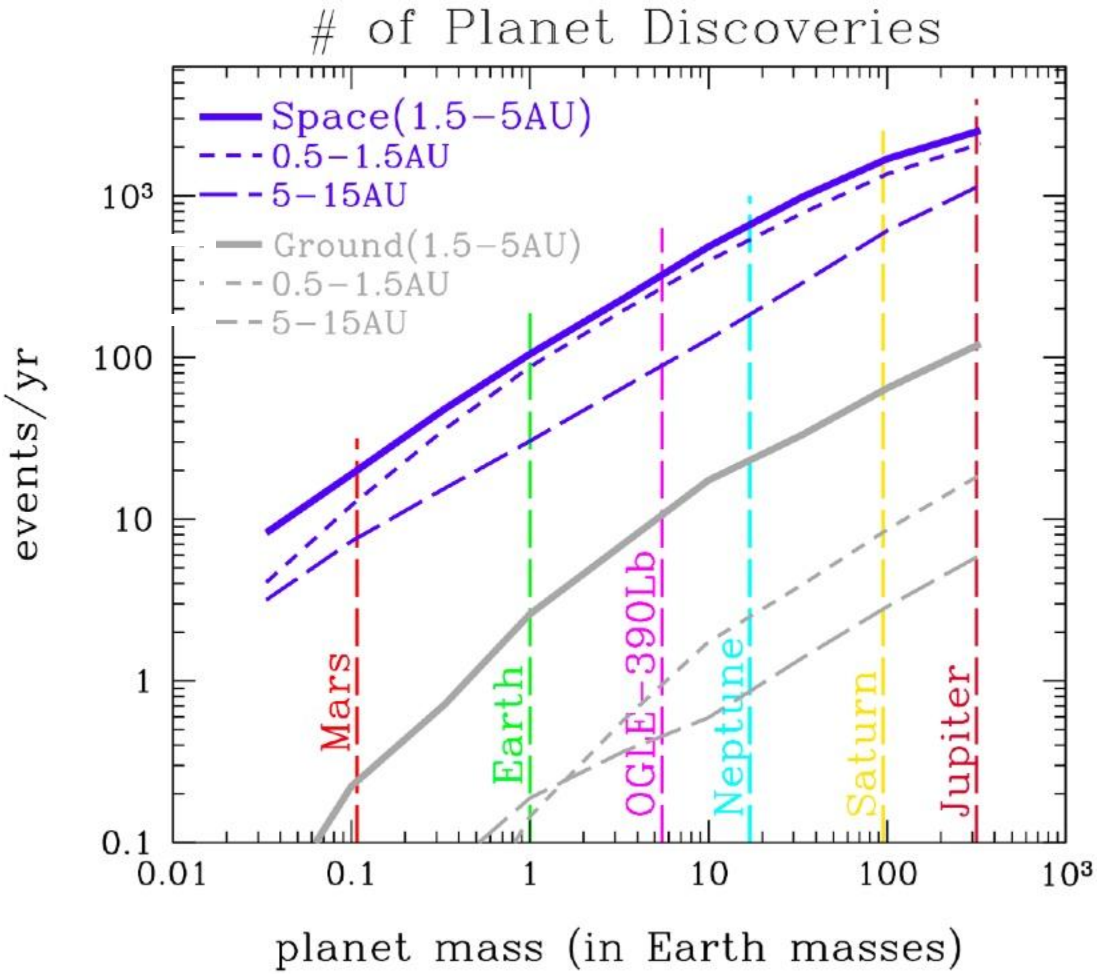


Optical

- Dust obscures the best microlensing fields toward the center of the Galaxy
- An infrared telescope unveils the stars shrouded in dust
- Most stars are M stars, which strongly radiate in the IR

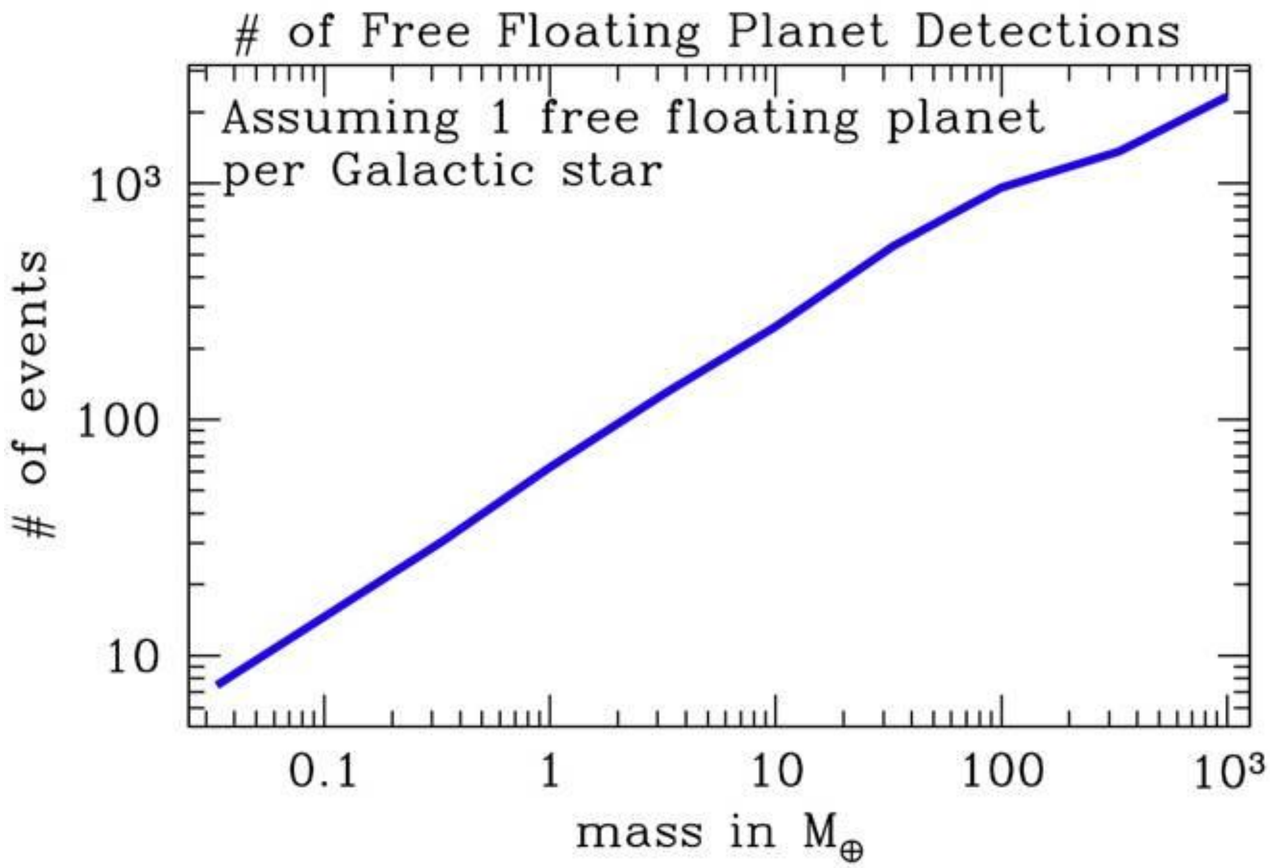
WFIRST (bound) exoplanet discoveries

- The number of expected WFIRST planet discoveries per 9-month observing season as a function of planet mass, assuming every star has a planet at the given mass.
- Microlensing is most sensitive to planets beyond the ‘snow line’, 1 to 5 AU.
- This is where planets are believed to form most efficiently.



Detection sensitivity rises with mass

WFIRST free-floating planets discoveries



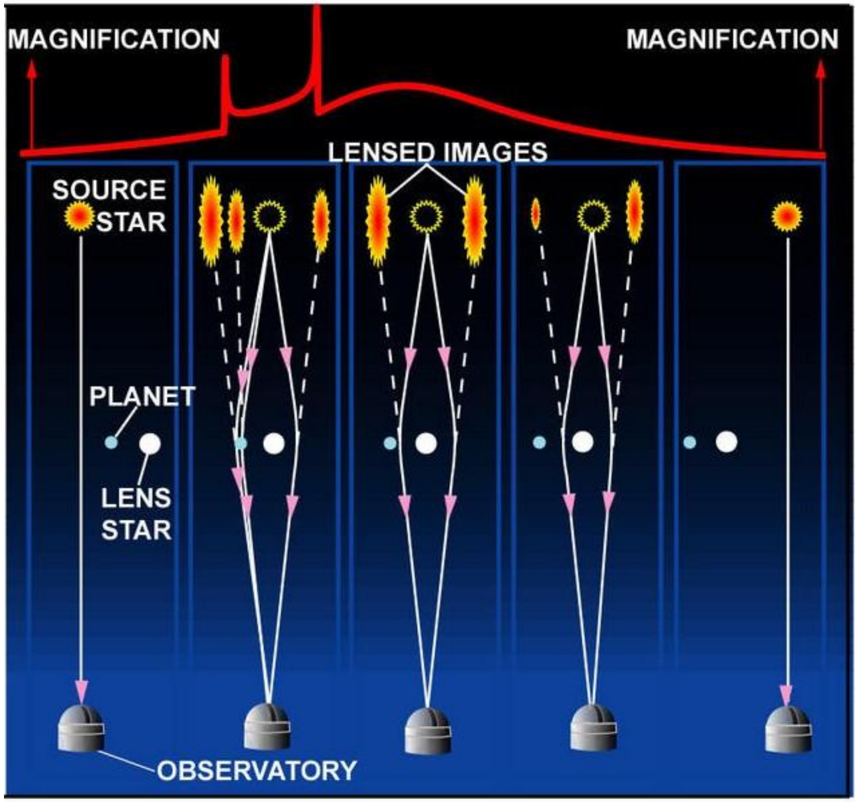
Detection sensitivity rises with mass

Exoplanet characterization

- What we measure:
  - Mass ratio of the exoplanet to the lensing star
  - Projected star-planet separation (in units of Einstein radius)
  - Angle between source trajectory and lens star-planet axis (in units of angular Einstein radius)
- If the lens star is observed, we can estimate its distance and mass. This determines the Einstein radius, and permits us to estimate absolute planet mass and star-planet separation
- In rare cases it's possible to solve the planet's orbit
- Lensing happens only once (except in 'exotic' cases); we don't get to come back and see the planet again.

Summary of Exoplanetary Microlensing

- Foreground 'lens' star + planet bend light of background 'source' star
- Multiple distorted images; total brightness change is the only observable
- Sensitive to planetary mass
- Low mass planet signals are rare, but not weak
- Stellar lensing probability toward galactic center is a few $\times 10^{-6}$
- Planetary lensing probability can range from 0.001 to 1 depending on event details
- Peak sensitivity for planets is at 2-3 AU, near the Einstein ring radius, R_E



Appendix

Scott Gaudi's microlensing movies

- **Single (point) lens**
- **Binary lens -- major image**
- **Binary lens -- minor image**

- Three parameters describe the planet orbiting the lens star
  - **Mass ratio** $q = M_{planet}/M_{star}$
  - **Projected separation** of star and planet in units of Einstein radius $d = s/R_E$, where s is projected separation
  - **Planet position angle** with respect to the lens-source axis
- These movies show how each parameter affects the observed microlensing event.

References

- 'Exoplanetary Microlensing' by Scott Gaudi, in Exoplanets, ed. Sara Seager, University of Arizona Press, 2010. Also at http://arxiv.org/abs/1002.0332
- 'Introduction to Gravitational Microlensing', by Shude Mao, http://arxiv.org/abs/0811.0441
- List of papers on exoplanet detection efficiencies with ground based microlensing data (provided by Dave Bennett):

  Rhie et al. (2000): first detection efficiency calculation - using a single event, but with sensitivity down to an Earth-mass. Uses a simple and relatively fast method: http://adsabs.harvard.edu/abs/2000ApJ...533..378R

  Gaudi & Sackett (2000): 1st paper on planetary detection efficiencies - laying out the method: http://adsabs.harvard.edu/abs/2000ApJ...528...56G

  Albrow et al. (2000): applying Gaudi method to a single event. http://adsabs.harvard.edu/abs/2000ApJ...535..176A

  Gaudi et al. (2002): Full analysis of 5 years of PLANET data: http://adsabs.harvard.edu/abs/2002ApJ...566..463G

  See also Albrow et al. (2001) http://adsabs.harvard.edu/abs/2001ApJ...556L.113A

  Bond et al. (2002): Limits on planets in 3 high magnification limits, including the first limits on planets orbiting a star in another galaxy. http://adsabs.harvard.edu/abs/2002MNRAS.333...71B

  Yoo et al. (2004): Limits on planets in a single high magnification event: http://adsabs.harvard.edu/abs/2004ApJ...616.1204Y

  Dong et al. (2006): Limits on planets in a single high magnification event: http://adsabs.harvard.edu/abs/2006ApJ...642..842D

  Sumi et al. (2010): A planet discovery paper, with a small section on relative detection efficiencies: http://adsabs.harvard.edu/abs/2010ApJ...710.1641S

  Gould et al. (2010): Analysis of a uniform sample of high magnification events: http://adsabs.harvard.edu/abs/2010ApJ...720.1073G